Exam. Code : 103201
Subject Code : 1026

B.A./B.Sc. 1st Semester

MATHEMATICS

Paper—II

(Calculus & Trigonometry)

Time Allowed—Three Hours] [Maximum Marks—50

Note :— Attempt **FIVE** questions in all selecting at least **ONE** question from each section. The **fifth** question may be attempted from any section.

SECTION—A

1. (a) Prove that between any two distinct real numbers, there is always an irrational number and therefore, infinitely many irrational numbers.

(b) Show that the function $f(x)=\begin{cases} \dfrac{x}{|x|+x^2}, & x \neq 0 \\ k, & x=0 \end{cases}$

cannot be made continuous at x = 0 regardless of the choice of k. 5,5

2. (a) Show that the set $A = \{(\sin x + \cos x)^2 : 0 \leq x \leq \pi\}$ is bounded. Also find g.l.b. and l.u.b. of A.

(b) Show that the function $f(x)=\dfrac{1}{x}$ is not uniformly continuous in (0, 1]. 5,5

SECTION—B

3. (a) Differentiate $\sin^{-1}(\tanh x^2)$ w.r.t. x.

(b) Prove that :

$$\frac{d^n}{dx}\left(\frac{\log x}{x}\right)=\frac{(-1)^n n!}{x^{n+1}}\left[\log x-1-\frac{1}{2}-\frac{1}{3}-\ldots\ldots-\frac{1}{n}\right].$$

5,5

4. (a) Evaluate :

$$\lim_{x\to 0+}(\cot x)^x.$$

(b) State and prove Taylor's theorem with Cauchy's form of remainder. 5,5

SECTION—C

5. (a) If $\sin(\theta + i\phi) = \tan\alpha + i\sec\alpha$, then show that $\cos 2\theta \cosh 2\phi = 3$.

(b) For $n \in Z$, prove that :

$$(\sqrt{3}+i)^n+(\sqrt{3}-i)^n=2^{n+1}\cos\frac{n\pi}{6}.$$ 5,5

6. (a) If $\cosh u = \sec\theta$, show that :

$$u=\log\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right).$$

(b) Show that the roots of the polynomial equation $(1 + x)^{2n} + (1 - x)^{2n} = 0$ are given by $x=i\tan\frac{(2k+1)\pi}{4n}$, where k = 0, 1, 2, ..., 2n – 1. 5,5

SECTION—D

7. (a) Prove that i^i is wholly real and find its principal value. Also show that the values of i^i form a G.P.

(b) Prove that :

$$\cos^7 \theta = \frac{1}{2^6}[\cos 7\theta + 7\cos 5\theta + 21\cos 3\theta + 35\cos \theta].$$

5,5

8. (a) Let S_n be the sum of n terms of the series $\sin \theta + \sin 2\theta + \sin 3\theta + \ldots\ldots.$ Prove that :

$$\lim_{n\to\infty} \frac{S_1 + S_2 + S_3 + \ldots\ldots + S_n}{n} = \frac{1}{2}\cot\frac{\theta}{2}.$$

(b) Use Gregory series to prove that :

$$\left(\frac{2}{3}+\frac{1}{7}\right) - \frac{1}{3}\left(\frac{2}{3^3}+\frac{1}{7^3}\right) + \frac{1}{5}\left(\frac{2}{3^5}+\frac{1}{7^5}\right) + \ldots\ldots = \frac{\pi}{4}.$$

5,5